Mixing Scenarios for Lattice String Breaking.

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Abstract

We present some simple scenarios for string breaking on the lattice based on a crude strong coupling model introduced previously. We review the dependence of the model on lattice spacing and extend it to include degenerate dynamical quarks and also meson exchange diagrams. A comparison is made between quenched and unquenched calculations. We examine string breaking in the presence of a static quark-diquark system, a situation that is specific to $SU(3)$.

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1 Introduction

String breaking has importance in lattice calculations because it reveals directly the effect of dynamical quarks. However the energy crossover effect characteristic of string breaking has proved difficult to observe [1]-[7]. Recently progress has been made and string breaking has been observed as a mixing phenomenon directly in $SU(2)$ Higgs models in three and four dimensions [8, 9, 10]. The phenomenon is currently under investigation in the much more demanding case of QCD [11, 12]. A crude model of the process based on strong coupling ideas has been proposed which pictures it as a mixing process between a string state and a two-meson state [13, 14]. It shows that because of its dependence on a mixing angle the Wilson loop may not reveal the string breaking energy crossover unless the mixing region is sufficiently broad and that in any case it is not by itself a satisfactory observable for revealing the crossover. This is consistent with numerical simulations [8, 9] which show the need for a suite of operators with members appropriate to both the string and two-meson states.

In this paper we develop the crude strong coupling model to discuss a number of simple mixing scenarios and use it to discuss the relationship between quenched and unquenched calculations. We take the opportunity to improve the model by including energy factors associated with the emission of light quarks from bound states. These factors are required in order to achieve results independent of the lattice spacing. They were missed in the original presentation [13] but are essential parameters [14, 15].

We elaborate the model on the one hand by including several quark flavours and on the other by including diagrams that represent meson exchange.

Finally we apply our approach to more complex situations that are specific to $SU(3)$ and examine the case of string breaking in the presence of a static quark-diquark system.

2 Rules for Simple Planar Model

The model we use was explained previously [13, 14]. It is based on simple planar lattice diagrams with internal dynamical quark loops. The initial formulation of the model is in terms of the rules for a “strong coupling expansion” [16, 17], modified to accommodate the presence of bound states in a simple way. The rules are (slightly modified from those of ref [13] as indicated in ref [14]):

1. A factor of $e^{-\sigma}$ for each plaquette, where σ is the (dimensionless) string tension.

2. A factor of
$$2\kappa\left(\frac{1+\gamma.e}{2}\right)$$
for each Wilson quark line in the direction of the unit vector e, interior to the diagram. Here κ is the standard quark hopping parameter.

3. A factor of
$$2\kappa'\left(\frac{1+\gamma.e}{2}\right)$$
for each Wilson quark line propagating in the direction of the unit vector e along a static anti-quark line. This creates the model for the static-light bound state meson.

4. A factor of $\sqrt{W}$ for each emission of a light quark from a meson state. Here W is a (dimensionless) energy parameter that determines the rate of emission of light quarks from the meson. Such a parameter is essential in order that the results of the model are independent of the lattice spacing.

5. A factor of $(-1/3)$ (in the case of $SU(3)$) for each internal quark loop.

6. A trace over the spin matrix factors for each internal quark loop.

In the above the hopping parameter, κ, reflects the light quark mass, $2\kappa = e^{-m_q}$, and the bound state hopping parameter, κ', determines the energy of the static meson, $2\kappa' = e^{-E_M}$. (In ref [13] E_M was the energy of a two-meson state and therefore had twice the value assigned here.)
If we apply the above rules to the diagram in Fig. 1 we obtain for the string-string transition amplitude the result

$$\mathcal{G}_{SS} = \left(e^{-\sigma R}\right)^{T_1}\frac{1}{\sqrt{6}}We^{-m_qR}\left(e^{-2E_M}\right)^{T_2}\frac{1}{\sqrt{6}}We^{-m_qR}\left(e^{-\sigma R}\right)^{T_3} \quad . \tag{1}$$

3 Lattice Spacing Dependence

The string breaking scenario considered previously involved a static quark and anti-quark separated by a (dimensionless) distance R. In the notation of previous papers [13, 14], we have

$$a = e^{-\sigma R} \quad , \quad b = e^{-2E_M} \quad , \quad c = \frac{1}{\sqrt{6}}We^{-m_qR} \quad . \tag{2}$$

The basic string state has an energy σR and the two-meson state an energy $2E_M$. As shown in ref [13] the propagating eigenstates have energies $E_\pm = -\log\lambda_\pm$ where

$$\lambda_\pm = \frac{1}{2}\left\{a + b \pm \sqrt{(a-b)^2 + 4abc^2}\right\} \quad . \tag{3}$$

To pick out the lattice spacing independent part we can treat all the dimensionless parameters as $O(a_L)$, where a_L is the lattice spacing. We then treat R and any time interval, T, involved as $O(a_L^{-1})$ and expand to the lowest significant order in a_L. We then find

$$E_\pm = \frac{1}{2}\left\{\sigma R + 2E_M \pm \sqrt{(\sigma R - 2E_M)^2 + \frac{2}{3}W^2 e^{-2m_q R}}\right\} \quad . \tag{4}$$

Having carried out this operation we can treat the above formula as referring to dimensionfull quantities. Note that it is essential to have the energy parameter W as a factor in the quantity c in order that this result holds true [14, 15].
Similar remarks apply to other quantities in the model. The mixing angle is

$$\tan\theta = \frac{-(a-b) + \sqrt{(a-b)^2 + 4abc^2}}{2\sqrt{ab}c} \quad . \tag{5}$$

Carrying out the expansion in small quantities we find

$$\tan\theta = \frac{\sigma R - 2E_M + \sqrt{(\sigma R - 2E_M)^2 + \frac{2}{3}W^2 e^{-2m_q R}}}{\sqrt{\frac{2}{3}}We^{-m_q R}} \quad . \tag{6}$$

The formulae in ref [13] for the mixing range ΔR and the energy split at maximal mixing, ΔE are correct when the energy parameter W is included. As indicated in ref [14], the results are

$$\Delta R = \pi\sqrt{\frac{2}{3}}\frac{W}{\sigma}e^{-m_q R} \quad , \quad \Delta E = \sqrt{\frac{2}{3}}We^{-m_q R} \quad . \tag{7}$$

The relation

$$\Delta R = \pi\frac{\Delta E}{\sigma} \tag{8}$$

is preserved. All these formulae may now be regarded as being explressed in physical parameters. The transition energy parameter is set by ΔE. Eq (8) is then a prediction for the mixing range in terms of the maximal energy split.

4 Quenched/Unquenched Comparison

An important reason for measuring string breaking is to identify the effect of dynamical fermions directly. It is therefore useful to consider the comparison between relevant quantities in the quenched and unquenched situations. A natural amplitude to measure is the transition amplitude between the string and the two-meson state [18]. The unquenched case was considered previously [13, 14]. The result was an amplitude of the form

$$\mathcal{G}_{MS}(T) = \sqrt{ab}\sin\theta\cos\theta(\lambda_+^{(T-1)} - \lambda_-^{(T-1)}) \quad , \tag{9}$$

where T is the full interval in the time direction associated with the measurement. The significance of this form is that the amplitude is suppressed outside the mixing region where the factor $\sin\theta\cos\theta$ vanishes. If we reexpress the formula in terms of physical parameters then we find

$$\mathcal{G}_{MS}(T) = \frac{1}{2}\left(\frac{\sqrt{\frac{2}{3}}We^{-m_qR}}{\sqrt{(\sigma R - 2E_M)^2 + \frac{2}{3}W^2e^{-2m_qR}}}\right)\left(e^{-E_+T} - e^{-E_-T}\right) \quad . \tag{10}$$

The corresponding quenched approximation can be described in our model as a sum over graphs of the form shown in Fig. 2. The resulting amplitude expressed in terms of the parameters a, b and c is

$$\mathcal{G}_{MS}^{(Q)}(T) = \sum_{T_1=1}^{T-1} b^{T_2}ca^{T_1} \quad , \tag{11}$$

where $T = T_1 + T_2$ .The result is

$$\mathcal{G}_{MS}^{(Q)}(T) = cab\frac{b^{T-1} - a^{T-1}}{b-a} \quad . \tag{12}$$

When translated into physical parameters we find

$$\mathcal{G}_{MS}^{(Q)}(T) = \frac{1}{2}\left(\frac{\sqrt{\frac{2}{3}}We^{-m_qR}}{\sigma R - 2E_M}\right)(e^{-2E_MT} - e^{-\sigma RT}) \quad . \tag{13}$$

The quenched and unquenched results are very similar in the region $|\sigma R - 2E_M| > \sqrt{\frac{2}{3}}We^{-m_qR}$. In the complementary region, the inner part of the mixing region on our definition, they exhibit very different behaviour. The unquenched amplitude exhibits the continuous crossover while the quenched amplitude shows an abrupt change from one exponential behaviour to the other at the crossover point. It would be extremely

interesting if this contrast in behaviour could be verified in a real simulation. Although our model rather oversimplifies the relationship of the quenched and unquenched calculations it is still possible that a measurement of the above transition amplitude in a quenched simulation would yield an estimate of the energy W that could stand as a prediction for the unquenched calculation.

5 Several Flavours

So far the model has been formulated only for a single flavour of light quark. The presence of several flavours of quark is accounted for by summing over the N_f flavour insertions of quark loop. This means that the model has $N_f + 1$ channels, namely the string channel and the N_f two-meson channels associated with the different flavours of quark. It is sufficient to consider the case $N_f = 2$. Following the same analysis as in the single flavour case and assuming that the light quarks are degenerate we easily see that the summation of relevant graphs is achieved by considering a 3×3 correlation matrix $G(T)$ that obeys the update equation

$$G(T+1) = AG(T) \quad , \tag{14}$$

where

$$A = \begin{pmatrix} a & ac & ac \\ bc & b & 0 \\ bc & 0 & b \end{pmatrix} \quad , \tag{15}$$

and we can assume that $G(T) = A^T$. It is immediately obvious that the two-meson state that is anti-symmetric under the interchange of flavours propagates with the factor $b^T = e^{-2E_M T}$. It does not mix with the string state. The symmetric state does mix with the string. If we choose the symmetric and antisymmetric states as basis states then the matrix A assumes the form

$$A = \begin{pmatrix} a & \sqrt{2}ac & 0 \\ \sqrt{2}bc & b & 0 \\ 0 & 0 & b \end{pmatrix} \quad , \tag{16}$$

The problem therefore remains a two-channel mixing problem of the same form as before with the minor change that $c \to \sqrt{2}c$. There is no need to carry the explicit calculation further since the results are obvious.

The general case of N_f flavours follows the same lines. Only the completely flavour symmetric combination of two-meson states mixes with the string. The mixing being

of a strength $\sqrt{N_f}$ times greater than the single flavour case. The remaining $N_f - 1$ two-meson channels orthogonal to the symmetric one remain unmixed.
If the light quarks are not flavour degenerate then to a first approximation the two-meson state corresponding to the lightest quark will show the first and strongest mixing with the string state since it lies at the lowest energy. Heavier quarks will mix at larger distances with decreasing strength. It would be interesting to test this obvious and plausible scenario in a full QCD calculation.

6 Extended Planar Model

It was implicit in the summation technique used previously that diagrams of the form shown in Fig. 3 were not included [13, 14]. To include them requires a new rule for the vertex associated with the emission of the light quark pair. It is

7. A factor $\sqrt{W_p}$ for the pair emission vertex, where W_p is a dimensionless energy parameter. Again this factor is necessary in order to eliminate explicit dependence of the results on the lattice spacing.

If, after emission, such a light quark-anti-quark pair were to propagate along a link, the corresponding factor according to the rules would be

$$\left(\frac{1+\gamma.e}{2}\right) \otimes \left(\frac{1-\gamma.e}{2}\right) (2\kappa)^2 \tag{17}$$

Naively the hopping parameter factor $(2\kappa)^2$ is equal to e^{-2m_q}. In the context of the model it would not be unreasonable to identify the quark pair exchange as a meson exchange. If the meson has a mass m then we should replace $(2\kappa)^2$ with e^{-m} . This is the interpretation we will adopt.
For completeness we give the list of rules for computing diagrams that include strings, two-meson states and light meson exchanges.

1. A factor $a = e^{-\sigma R}$ that propagates the string by one time step.

2. A factor $b = e^{-2E_M}$ that propagates the two-meson state by one time step.

3. A factor $c = (W/\sqrt{6})\,(2\kappa)^R$ associated with the transition from string to two-meson state and vice-versa.

4. A factor $\delta = \frac{1}{6} W_p e^{-mR}$ for each quark-anti-quark exchange.

As in the original model the summation can be carried out by computing a 2×2 matrix of propagators connecting the string and two-meson states

$$G(T) = \left(\begin{array}{cc} G_{SS}(T) & G_{SM}(T) \\ G_{MS}(T) & G_{MM}(T) \end{array} \right) \quad , \tag{18}$$

which obeys the update equation

$$G(T+1) = AG(T) \quad , \tag{19}$$

where

$$A = \left(\begin{array}{cc} a & ac \\ bc & b(1+\delta) \end{array} \right) \quad , \tag{20}$$

The natural solution to eq(19) is $G(T) = A^T$. As before A can be expressed in the form $A = DO\Lambda O^{-1}D^{-1}$, where D and O have the forms

$$D = \left(\begin{array}{cc} \sqrt{a} & 0 \\ 0 & \sqrt{b} \end{array} \right) \quad , \quad O = \left(\begin{array}{cc} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{array} \right) \quad , \tag{21}$$

and Λ is a diagonal matrix of eigenvalues,

$$\lambda_\pm = \frac{1}{2}\left\{ (a + b(1+\delta)) \pm \sqrt{(a - b(1+\delta))^2 + 4abc^2} \right\} \quad . \tag{22}$$

The mixing angle θ, that describes the overlap of the string and two-meson channels with the eigenchannels of definite energy, $E_\pm = -\log\lambda_\pm$, satisfies

$$\tan\theta = \frac{-(a - b(1+\delta)) + \sqrt{(a - b(1+\delta))^2 + 4abc^2}}{2\sqrt{ab}\; c} \quad . \tag{23}$$

The analysis of mixing is essentially the same as before except that the centre of the mixing region where $\theta = \pi/4$ occurs when $a = b(1+\delta)$ that is when $R = R_c$ where

$$\sigma R_c = 2E_M + \log(1+\delta) \simeq 2E_M + \frac{1}{6}W_p e^{-mR_c} \quad . \tag{24}$$

This is consistent with the idea that the extra light meson exchange represented by δ corresponds to a potential interaction between the two static mesons of the form $(W_p/6)e^{-mR}$ that displaces the centre of the mixing region in the appropriate way. If we estimate the mixing region as $\Delta R = \frac{\pi}{2}\frac{dR}{d\theta}|_{R=R_c}$ then we find

$$\Delta R = \pi\sqrt{\frac{2}{3}}\frac{We^{-m_q R_c}}{\sigma - \frac{1}{6}mW_p e^{-mR_c}} \quad . \tag{25}$$

This is essentially the same result as derived previously [13, 14] but now including the effect of the potential between the mesons. As before the energy split at the point of maximum mixing is $\Delta E = 2c$, that is

$$\Delta E = \sqrt{\frac{2}{3}} W e^{-m_q R} \quad . \tag{26}$$

We have therefore the relation between ΔR and ΔE of the form

$$\Delta R = \pi \frac{\Delta E}{\sigma - \frac{1}{6} m W_p e^{-m R_c}} \quad . \tag{27}$$

These results may now be interpreted as referring to quantities in physical units. In deriving them we have dropped corrections that vanish with the lattice spacing.

7 Quark/Diquark String Breaking

So far string formation and breaking in QCD has been sought in a context in which the string joins a static quark and anti-quark pair. An interesting alternative scenario involves a static-light diquark supporting a string along with another static quark. This is a situation that is particular to $SU(3)$ gauge theory. We again have quenched and unquenched possibilities.

A relevant correlation function can be formulated in the following way.

$$\mathcal{F}(T) = \langle \frac{1}{6} U(C_1)_{\alpha_1 \beta_1} U(C_2)_{\alpha_2 \beta_2} G_{q \alpha_3 \beta_3}(T) \epsilon_{\alpha_1 \alpha_2 \alpha_3} \epsilon_{\beta \beta \beta} \rangle \tag{28}$$

Here the angle brackets indicate averaging over the gauge fields and $G_{q\alpha_3\beta_3}(T)$ is the light quark propagator from the origin to the origin over a time interval T . The matrices $U(C_1)$ and $U(C_1)$ are each the product of gauge $SU(3)$ matrices along the paths C_1 and C_2 that contain the static quarks and join the ends of the light quark propagator, as indicated in Fig. 4.

In eq(28) we have suppressed the spin labels of the quarks. However a feature of diquark dynamics is its sensitivity to the spins of the constituent quarks [19]. For this reason it is important in this case to take account of the static quark spins as well as that of the light quark. We will do this by assuming that the static quark and light quark bind in a singlet state. The triplet state, being higher in mass [19], we will ignore for simplicity. In that case lines in our diagramatic model that carry a diquark state will have simple scalar propagators while the lines carrying a light or static quark in

the direction e will have the standard spin factor $(1+\gamma.e)/2$ as well as the appropriate energy exponential.
A typical diagram that contributes to the amplitude in our simplified planar diagram model is shown in Fig. 4. The light quark propagates along the static quark lines in a diquark combination and also makes transitions between the static quarks on each side of the diagram. This diagram is part of the quenched approximation. The incorporation of dynamical quarks into the model results in diagrams such as that in Fig. 5 where internal quark loops appear creating intermediate states that involve a static nucleon, containing one static and two light quarks, together with a static meson, containing a static quark and a light anti-quark.

7.1 Quenched Calculation

Because the light quark can make a transtion from one static quark to the other, even the quenched calculation is not trivial. Within the model we are concerned with two channels. The left channel in which the light quark propagates along the static quark at the origin and the right channel in which it propagates along the static quark displaced a distance R from the origin. We will associate a hopping parameter κ'' with the motion of the light quark along a static quark line. We will introduce an energy W_d that provides a factor $\sqrt{W_d}$ for the strength of emission of the light quark from a static diquark. With these rules the contribution to $\mathcal{F}(T)$ of the diagram in Fig. 4 is

$$\left(2\kappa'' e^{-\sigma R}\right)^{T_1} \left(\frac{W_d}{2} e^{-m_q R}\right) \left(2\kappa'' e^{-\sigma R}\right)^{T_2} \left(\frac{W_d}{2} e^{-m_q R}\right) \left(2\kappa'' e^{-\sigma R}\right)^{T_3} \left(\frac{1+\gamma_0}{2}\right) \tag{29}$$

The factors of $\frac{1}{2}$ that accompany the passage of the light quark across the diagram follow from the γ-matrix algebra. Because it plays no further rôle from now on we will drop the quark spin factor in $\mathcal{F}(T)$. In order to sum up diagrams of the type we have just evaluated we require a 2×2 matrix of propagators in order to describe the two channels in the problem. We have

$$G(T) = \begin{pmatrix} G_{LL}(T) & G_{LR}(T) \\ G_{RL}(T) & G_{RR}(T) \end{pmatrix} \quad . \tag{30}$$

Up tp terms that vanish with the lattice spacing, $\mathcal{F}(T) = G_{LL}(T)$. The summation over diagrams proceeds by requiring the propagators to obey the stepping relation

$$G(T+1) = A' G(T) \quad , \tag{31}$$

where

$$A' = \begin{pmatrix} a' & a'c' \\ a'c' & a' \end{pmatrix} \quad , \tag{32}$$

where $a' = 2\kappa'' e^{-\sigma R} = e^{-\sigma R - E_d}$ and $c = \frac{1}{2} W_d e^{-m_q R}$ and we have identified $E_d = -\log(2\kappa'')$ as the static diquark energy. It is immediately obvious that there are two eigenchannels. A symmetrical superposition of the left and right channels and an antisymmetric superposition.

Choosing these as the basis channels A' takes the form

$$A' = \begin{pmatrix} a'(1+c') & \\ & a'(1-c') \end{pmatrix} \tag{33}$$

The eigenenergies of these channels respectively $\epsilon_{\pm} = -\log(a'(1 \pm c')) \simeq E_d + \sigma R \pm \frac{1}{2} W_d e^{-m_q R}$. We see here the influence of the quark exchange in splitting the degeneracy of the two string-diquark channels.

7.2 Dynamical Quarks

The effect of dynamical quarks is to insert light quark loops into the diagrams. As pointed out above this creates the possibility of two new channels one with a static nucleon at the origin and a static meson at a distance R and the other with the nucleon and meson interchanged. See Fig 5 .

In order to sum over the diagrams of the model we need a 4×4 matrix of correlators.

$$G(T) = \{G_{ij}(T)\} \quad , \tag{34}$$

where $i, j = 1, 2, 3, 4$ and the labels 1 and 2 refer to the diquark-string channels with the diquark on the left and right respectively (L and R above). The labels 3 and 4 refer to the nucleon-meson channels with the nucleon on the left and right respectively.

The upgrade step is

$$G(T+1) = PG(T) \quad , \tag{35}$$

where now

$$P = \begin{pmatrix} a' & a'c' & a'c'' & 0 \\ a'c' & a' & 0 & a'c'' \\ b'c'' & 0 & b' & 0 \\ 0 & b'c'' & 0 & b' \end{pmatrix} \quad , \tag{36}$$

where a' and c' are as before and

$$b' = e^{-(E_N + E_M)} \quad \text{, and} \quad c'' = \frac{\sqrt{WW_N}}{\sqrt{6}} e^{-m_q R} \quad . \tag{37}$$

Here W_N is the energy parameter associated wiith the emission of a light quark by a static nucleon. The mixing phenomena in the model are easily disentangled because the left-right symmetry of P means that the symmetric diquark-string state mixes only with the symmetric nucleon-meson state and similarly for the corresponding antisymmetric states. If we choose these symmetric and anti-symmetric states as the basis channels then $G(T)$ and P have the form

$$G(T) = \begin{pmatrix} G^{(S)}(T) & 0 \\ 0 & G^{(A)}(T) \end{pmatrix} \quad , \quad P = \begin{pmatrix} P^{(S)} & 0 \\ 0 & P^{(A)} \end{pmatrix} \quad , \tag{38}$$

and

$$P^{(S)} = \begin{pmatrix} a'(1+c') & a'c'' \\ b'c'' & b' \end{pmatrix} \quad , \quad P^{(A)} = \begin{pmatrix} a'(1-c') & a'c'' \\ b'c'' & b' \end{pmatrix} \quad . \tag{39}$$

We have $G^{(S)}(T+1) = P^{(S)}G^{(S)}(T)$ and $G^{(A)}(T+1) = P^{(A)}G^{(A)}(T)$ with the solutions $G^{(S)}(T) = \left(P^{(S)}\right)^T$ and $G^{(A)}(T) = \left(P^{(A)}\right)^T$. We can write

$$P^{(S,A)} = DO^{(S,A)}\Lambda^{(S,A)}(O^{(S,A)})^{-1}D^{-1} \quad , \tag{40}$$

where

$$D = \begin{pmatrix} \sqrt{a'} & 0 \\ 0 & \sqrt{b'} \end{pmatrix} \quad , \quad O^{(S,A)} = \begin{pmatrix} \cos\theta_{S,A} & -\sin\theta_{S,A} \\ \sin\theta_{S,A} & \cos\theta_{S,A} \end{pmatrix} \quad . \tag{41}$$

The eigenvalues of $P^{(S,A)}$ are the columns of $DO^{(S,A)}$ and the eigenvalues are the entries in the diagonal matrix

$$\Lambda^{(S,A)} = \begin{pmatrix} \lambda_+^{(S,A)} & 0 \\ 0 & \lambda_-^{(S,A)} \end{pmatrix} \quad . \tag{42}$$

It is easily established that

$$\lambda_\pm^{(S)} = \frac{1}{2}\left\{a'(1+c') + b' \pm \sqrt{(a'(1+c') - b')^2 + 4a'b'c''^2}\right\} \tag{43}$$

and

$$\lambda_\pm^{(A)} = \frac{1}{2}\left\{a'(1-c') + b' \pm \sqrt{(a'(1-c') - b')^2 + 4a'b'c''^2}\right\} \tag{44}$$

The mixing angles are given by

$$\tan\theta_S = \frac{-(a'(1+c') - b') + \sqrt{(a'(1+c') - b')^2 + 4a'b'c''^2}}{2\sqrt{a'b'}c''} \quad , \tag{45}$$

and a similar formula for $\tan\theta_A$ where $c' \to -c'$. The mixing analysis is very similar to that for the original string breaking model and leads to the results that the critical value

of the separation in the symmetric and antisymmetric cases are given by $b' = a(1 \pm c')$, that is

$$\sigma R_c = E_N + E_M - E_d \pm \frac{1}{2} W_d e^{-m_q R_c} \quad , \tag{46}$$

and the mixing ranges are

$$\Delta R_{S,A} = \pi \sqrt{\frac{2}{3}} \frac{\sqrt{W W_N} e^{-m_q R_c}}{\sigma \pm \frac{1}{2} m_q W_d e^{-m_q R_c}} \quad , \tag{47}$$

and the energy splits at maximum mixing are

$$\Delta E_{S,A} = \sqrt{\frac{2}{3}} \sqrt{W W_N} e^{-m_q R_c} \quad . \tag{48}$$

Although the energies are likelier to be higher than in the standard string scenario the static quark-diquark string is also of great interest as an alternative string system specific to $SU(3)$ and because of what it can reveal about diquark dynamics.

8 Conclusions

We have explored further the consequences of a simple picture of string breaking as a mixing phenomenon. We have clarified the elimination of lattice spacing dependence in the model by improving it over its original formulation with the introduction of appropriate energy factors for quark and meson emission from bound states. We have shown that it is possible to incorporate a meson-meson potential into the the original model and find, as might be expected, that it does not lead to radically different results on mixing.

We have also investigated the effect of degenerate light quark flavours. Here the only significant effect is a slight strengthening of the mixing effect by a factor $\sqrt{N_f}$. It is also interesting that only the totally symmetric combination of two-meson channels participates in string breaking leaving the orthogonal combinations unmixed.

We point out that a comparison of the quenched and unquenched calculations of string breaking is of considerable interest as a simulation. The appropriate measurement is of the string-meson transition amplitude. On the basis of the model we show that the two cases are likely to be rather similar in the outer part of the mixing region but show distinct behaviours in the central part of the mixing region, where the quenched amplitude shows a sharp transition between energy exponentials and the unquenched case shows a more gradual crossover. It is possible, if the comparison between the

two cases is made in an appropriate way in an actual simulation, that the quenched measurement could provide a prediction for the energy split and the size of the mixing region in the unquenched case.

We study static quark-diquark string breaking which shows some simple but interesting patterns of channel mixing. The basic phenomenon is very similar to that of the quark-anti-quark string breaking. The symmetry of the system however provides us with two channels in the quenched case. These each go on to mix with appropriately symmetrical or antisymmetrical meson-nucleon channels. This pattern of double string breaking is interesting in itself and more so since it is specific to the case of $SU(3)$. It seems worth investigating this case even though the higher energies involved will make it harder to measure. Perhaps an approach using a non-symmetric lattice with a fine lattice spacing the time direction would be appropriate in this case.

The success of simulations using the $SU(2)$ Higgs model in observing string breaking [8, 9] suggests that similar results could be obtained for the scenarios discussed in this paper with scalar matter fields replacing the dynamical quarks. It would of course be necessary to go to the $SU(3)$ Higgs model to see the effects discussed in the previous section.

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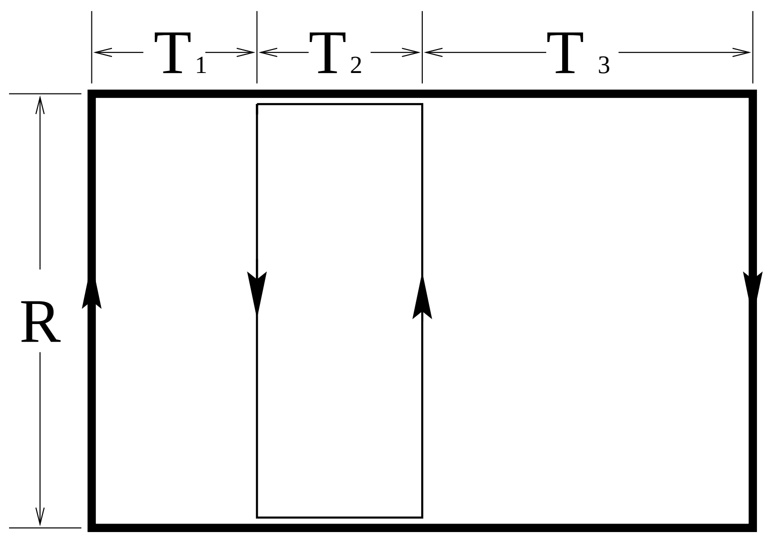

Figure 1: Wilson loop (heavy line) containing internal quark loop (light line).

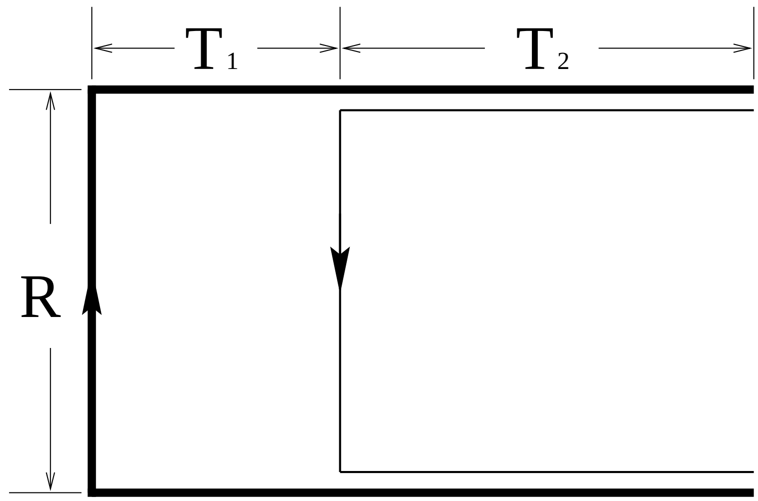

Figure 2: Contribution to the transition amplitude in the quenched approximation.

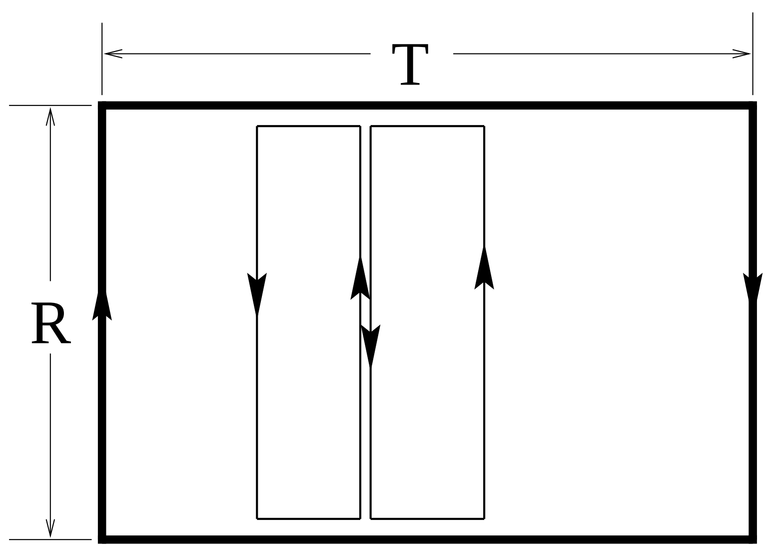

Figure 3: Wilson loop contribution with a light quark-anti-quark exchange.

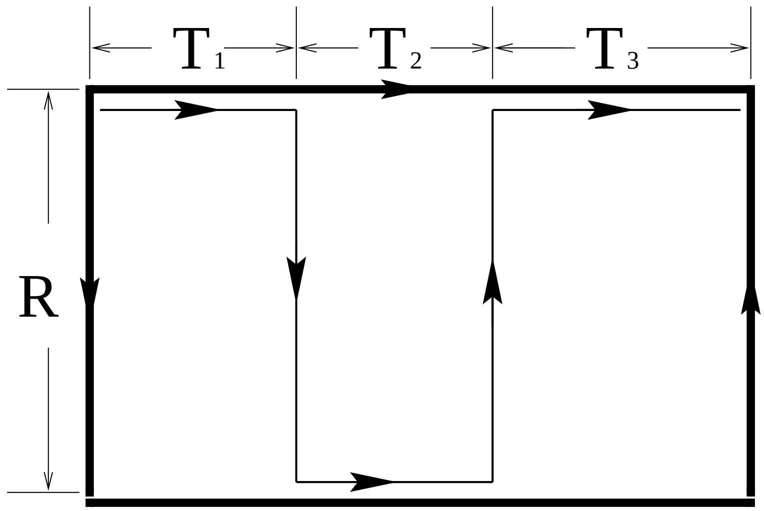

Figure 4: Contribution to a heavy quark-diquark amplitude without internal quark loops. The path C_1 runs along the top of the graph. The path C_2 runs down the left side along the bottom and up the right side of the graph.

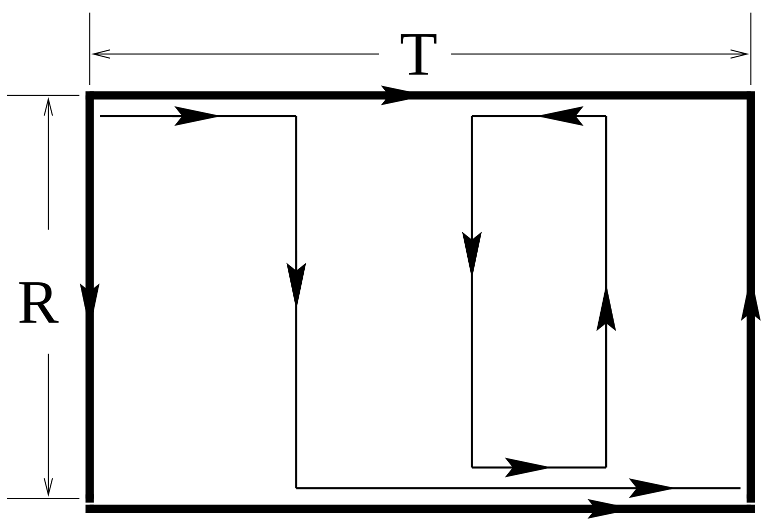

Figure 5: Contribution to a heavy quark-diquark amplitude with an internal quark loop.